A Gaussian Model Approach for the Prediction of Speckle Reduction with Spatial and Frequency Compounding

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Abstract - In the past different analytical expressions have been derived to describe the correlation curve $\rho(\Delta x)$ of two ultrasonic images which are created from a set of subapertures for spatial compounding or a set of subbands for frequency compounding.

We show that both speckle reduction with spatial compounding (SC) and with frequency compounding (FC) can be modeled by a Gaussian shaped correlation curve of the kind $\rho(\Delta x) = \exp(-\Delta x^2/\alpha)$, where Δx represents either the relative shift between neighboring subapertures or neighboring subbands and the parameter α is a constant which describes the inherent properties of the two compounding algorithms.

With our experiments the parameter α is estimated individually for SC and FC and the model is used to determine the optimum number of subimages to obtain maximum lesion detectability with a predefined loss in resolution. The results are compared with experimental data.

INTRODUCTION

The signal-to-noise ratio (SNR) of an ultrasound pure speckle image is typically defined as the ratio of the image brightness mean μ and the image brightness variance σ:

$$\mathrm{SNR} = \frac{\mu}{\sigma} = (\sum_{k=1}^{K} X_k) / (\sqrt{\sum_{k=1}^{K} X_k^2 - \left(\sum_{k=1}^{K} X_k\right)^2}) \quad (1)$$

where the index k denotes the K pixels of the video image X.

To increase the signal-to-noise ratio (SNR) of ultrasound images by the means of compounding two basic approaches can be pursued, i.e. either compounding images that are recorded from different scan directions with spatial compounding (*SC*), or compounding images with different frequency contents using subbands of the image frequency spectrum with frequency compounding (*FC*). Both approaches are well documented in the literature, e. g. [1], [5], [7].

The theoretical improvement in SNR that is achievable with compounding can be related to the cross-correlation coefficients ρ_{ij} of the N subimages as derived in [2]:

$$\frac{\mathrm{SNR}_N}{\mathrm{SNR}_1} = 1 / \sqrt{\frac{1}{N^2} \sum_{i,j} \rho_{ij}} = 1 / \sqrt{\frac{1}{N} \{1 + \frac{2}{N} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{ij}\}} \overset{\mathrm{def}}{=} \frac{1}{C(N)} \quad (2)$$

where N denotes the number of the compounded subimages. C(N) is the so called speckle contrast and the cross-correlation coefficients r_{ij} of the subimages $X^{(i)}$ and $X^{(j)}$ can be computed by:

$$\rho_{ij} = \frac{\sum_{k=1}^{K} (X_k^{(i)} - \mu^{(i)})(X_k^{(j)} - \mu^{(j)})}{\sqrt{\sum_{k=1}^{K} (X_k^{(i)} - \mu^{(i)})^2 \sum_{k=1}^{K} (X_k^{(j)} - \mu^{(j)})^2}} \quad (3)$$

The obtained improvement in SNR is often described by the effective number of statistically independent images N_{eff} that are contributing to the compounded image:

$$N_{eff} = \left(\frac{\mathrm{SNR}_N}{\mathrm{SNR}_1}\right)^2 = \frac{1}{C(N)^2} \quad (4)$$

Note that if the N subimages which are used for the compounding procedure are statistically independent, N_{eff} will be equal to N which denotes the theoretical limit.

For SC and FC the cross-correlation coefficients are a function of the relative shift Δx, where Δx either refers to a shift in aperture or a shift in center frequency of the subbands. Introducing $\rho_{ij} = \rho(|i-j|\Delta x)$ into Equation (2) the speckle contrast C(N) can be simplified as follows:

$$C^2(N,\Delta x) = \frac{1}{N}\left[1 + \frac{2}{N}\sum_{i=1}^{N-1}(N-i)\cdot\rho(i\Delta x)\right] \qquad (5)$$

In the following the function ρ(Δx) will be referred to as the correlation curve.

To optimize lesion detectability (LD) we adopt a measure which was originally introduced by Wagner *et al* [3] and was modified for SC by O'Donnell *et al* [2] in the following form:

$$LD = c\sqrt{\frac{1}{r\cdot C^2(N)}} = c\sqrt{\frac{N_{eff}}{r}} \qquad (6)$$

Here *c* is a constant referring to the geometry of the lesion and *r* serves as a relative measure for the degradation of the point spread function referred to in the following as the resolution loss. In our case *r* can be defined as the bandwidth to subbandwidth ratio in the axial direction for FC or, correspondingly, the ratio of aperture to subaperture width in the lateral direction for SC.

Note that the relative shift Δx can be expressed as a function of the resolution loss r and the number of subimages *N* as follows:

$$\Delta x(N,r) = \begin{cases}(r-1)/(N-1) & \text{for } N>1\\ 0 & N=1\end{cases} \qquad (7)$$

For a given correlation function ρ(Δx) and a predefined loss in resolution r the optimum number of subimages N_{opt} that will maximize the lesion detectability can be determined with Equation (6) and Equation (7) by minimizing the square of the speckle contrast C(N):

$$N_{opt} = \arg\min_N \left\{ C^2(N,\Delta x(N,r)) \right\} \qquad (8)$$

Note that $C^2(N)$ is equal to the inverse of the effective number of subimages N_{eff} as defined in Equation (4). Therefore Equation (8) will also maximize the effective number of subimages N_{eff}.

With knowledge of the correlation curve Equation (8) can easily be evaluated numerically and the problem of maximizing the lesion detectability reduces to adequately identify the correlation curve.

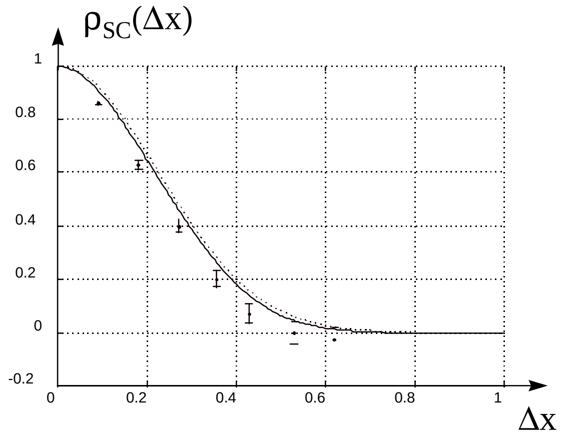

Figure 1: Theoretical SC correlation curve $\rho_{SC}(\Delta x)$ [-] as described by Equation (9), its Gaussian approximation [..] with α_{SC} = 0.1 and ρ_∞ = 0 (Equation (10)), and experimental data of Trahey *et al* [4]. Δx is the relative aperture shift between neighboring subapertures.

To describe ρ(Δx) different functions have been suggested in the literature (e.g. [1], [2], [7]). Wagner *et al* [7] derive an expression for the correlation curve of two demodulated video images:

$$\rho(\Delta x) = c\cdot\left|FT\left\{|p(\tilde{x})|^2\right\}\right|^2 \qquad (9)$$

with $p(\tilde{x})$ being the lateral or axial component of the point spread function and *c* a constant to assure $\rho(0)=1$. *FT* denotes the fourier transform. If the transformation variable $\tilde{x}$ is appropriately normalized, Δx can be considered to be either a relative shift in aperture or in center frequency independent of the characteristics of the experimental set-up [7]. Therefore we propose to model both the spatial correlation curve and the frequency correlation curve by a Gaussian function of the form $\rho(\Delta x) = \exp(-\Delta x^2/\alpha)$, where the parameter α is a constant which describes the inherent properties of the two compounding algorithms. To our knowledge, this approach has not been pursued in the literature.

If the subimages contain structural information the correlation curve will approach a non-zero value ρ_∞ for Δx approaching infinity. In this case it can be shown that ρ(Δx) can be rescaled as:

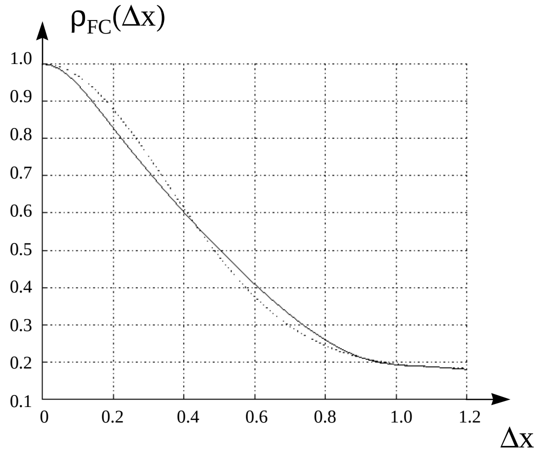

Figure 2: Experimentally determined FC correlation curve $\rho_{FC}(\Delta x)$ (-) for two subbands with a 2 MHz bandwidth at -6 dB and its Gaussian approximation (¨) with $\alpha_{FC} = 0.25$ and $\rho_\infty = 0.17$. Δx corresponds to the relative shift of the center frequencies of neighboring subbands

$$\rho(\Delta x) = \rho_\infty + (1 - \rho_\infty)\exp(-\Delta x^2 / \alpha) \qquad (10)$$

METHODS

For SC the correlation curve is very close to a Gaussian curve of the above kind since $\rho(\Delta x)$ can be described analytically by the convolution of two triangular functions in the far field [4]. The estimation of α_{SC} using a least square error criteria yields $\alpha_{SC} = 0.1$ as shown in Figure 1.

With our experiments we focused on the estimation of the parameter α_{FC}. 225 lines of RF-data were acquired with a Siemens Q2000 Scanner and a 5 MHz linear phased array at a sampling rate of 30.2 MHz. The relative -6 dB bandwidth of the probe was approximately 65 %. The system was set to B-mode with a single transmit focus at about 80 mm to achieve homogeneous resolution throughout the imaging plane. The region of interest was a square patch of approximately 30 mm x 30 mm at a depth of 55 mm. A custom-made tissue mimicking phantom was used which included a 1.5 dB circular positive contrast area of approximately 8 mm in diameter. The phantom was made from agar with added graphite powder and glass spheres to provide tissue like absorption and scattering.

The acquired image spectrum was divided into subbands by equiripple filters with

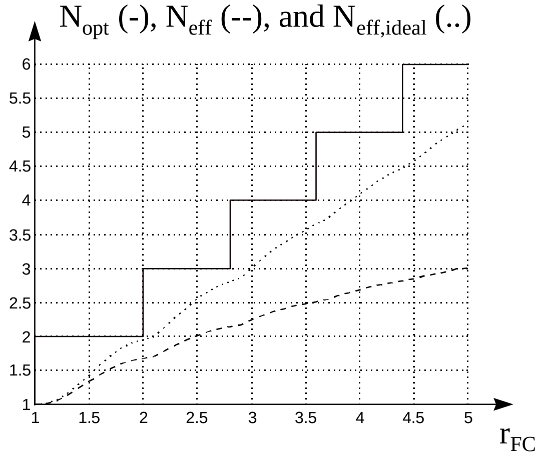

Figure 3: N_{opt} (-), N_{eff} (--) with $\alpha_{FC} = 0.27$ and $\rho_\infty = 0.17$, and $N_{eff,ideal}$ (¨) with $\alpha_{FC} = 0.27$ and $\rho_\infty = 0$ as a function of the resolution loss r_{FC} (= bandwidth to subbandwidth ratio for a FC image. The dotted line indicates the theoretical limit for N_{eff}.

transition bandwidths of approximately 0.5 MHz. The filters had a constant filter length of 120 taps which yielded stopband suppressions of approximately -40 dB. We found that the estimate of α_{FC} is not susceptible to the filter length and shape as long as the stopband suppression is below -30 dB. Two or more subbands of equal bandwidth were cut out of the image spectrum and moved from a complete overlap to a non-overlapping position. Subsequently the cross-correlation coefficients of the demodulated subimages were calculated as a function of the relative band shift Δx to identify the parameter α_{FC} and to experimentally determine the correlation curves for a varying number of subbands.

RESULTS

Figure 2 shows a typical correlation curve of two compounded subimages obtained from subbands with a bandwidth of 2 MHz at -6 dB. Note that a relative band shift of 0 refers to a complete overlap of the subbands and a shift of 1 refers to the -6 dB intersection of the subbands at the center frequency of the original image spectrum. The Gaussian approximation with $\alpha_{FC} = 0.25$ and $\rho_\infty = 0.17$ is indicated by a dotted line. For varying bandwidths an average value for α_{FC} could be determined of approximately 0.27.

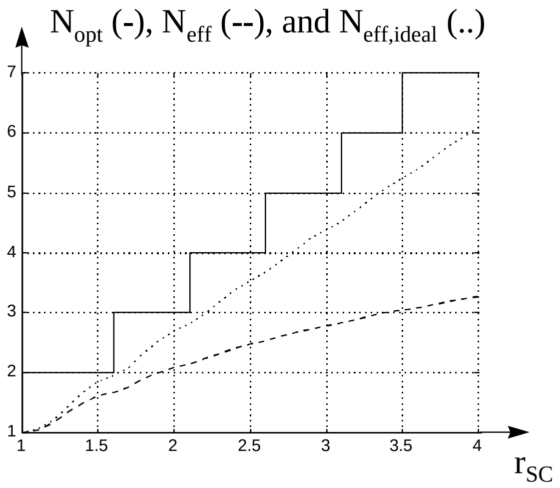

Figure 5: N_{opt} (-), N_{eff} (--) with $\alpha_{SC} = 0.1$ and $\rho_\infty = 0.17$, and $N_{eff,ideal}$ (--) with $\alpha_{SC} = 0.1$ and $\rho_\infty = 0$ r_{SC} (= aperture to subaperture width ratio) for a spatial compounded image. The dotted line indicates the theoretical limit for N_{eff}.

The identified parameter α_{FC} was used to calculate the optimum number of subbands N_{opt} which maximizes the LD coefficient using Equation (8). In Figure 3 we show N_{opt} and the corresponding effective number of images N_{eff} for the ideal and the experimental case. Note that the square root of N_{eff} is proportional to the LD coefficient given in Equation (6) and proportional to the square of the SNR improvement obtained by compounding. The curves for N_{eff} depend on the structural information that is present in the subimages expressed by ρ_∞. However, we can determine exactly where a change from n to $n + 1$ subbands is optimal to obtain maximum lesion detectability since N_{opt} does not change for varying ρ_∞. This result offers valuable information for the design of a FC imaging system.

Further we conclude from Figure 3 that the magnitude of N_{eff} never exceeds the magnitude of the resolution loss r, and therefore the LD coefficient can not be increased with frequency compounding (compare with Equation (6)).

Note that for FC a change from two to three subbands corresponds exactly to a resolution loss of 2 which is equal to a relative subband shift of 1 and a subbandwidth of half the image spectrum. This is in contrast to the results that we obtain for spatial compounding which is

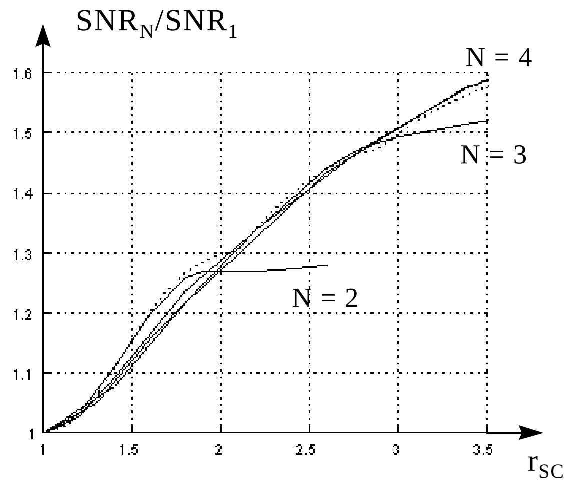

Figure 4: The SNR improvement for frequency compounded images with *modeled* (¨) and *measured* (-) curves; N indicates the number of subbands used for the compounding procedure. The relative measurement error is approximately 4 %.

illustrated in Figure 5. For SC the change from two to three subapertures for maximum lesion detectability corresponds to a resolution loss of approximately 1.7. This is equivalent to a relative aperture shift of 0.7. For the ideal case ($\rho_\infty = 0$) we find an improvement in lesion detectability of approximately $\sqrt{2}$. The reason for this improvement is that the correlation curve for SC given in Figure 1 is almost 0 for a relative aperture shift of 0.7 which is in contrast to the FC correlation curve.

Our results for FC and SC support findings of Trahey *et al* [3] who conclude that SC is preferable over FC when applying compounding algorithms for speckle reduction.

In Figure 4 we compare the theoretical results from the identification of the parameter α_{FC} with the results from our experiments. We find that within the relative measurement error the measured curves correspond very well to the predicted values indicated by a dotted line.

DISCUSSION

To deal with the difficulties of image quality assessment and lesion detectability measures, we have proposed a new approach to optimize speckle reduction by compounding. This approach is based on the constraint of a user-defined resolution loss and the subsequent computation of the optimal number of bands or subapertures for maximum lesion detectability.

Our experiments verify that for FC the improvement in lesion detectability or correspondingly the improvement in image SNR can be predicted by the proposed Gaussian model. For SC better results are expected. Note that the results do not depend on the characteristics of the experimental set-up.

Modeling the effects of SC and FC we found that the LD coefficient can not be improved by FC. However, the amount of speckle in the original image will be reduced correspondingly to the improvement in SNR. As a result the subjective impression of the image may well be enhanced and the use of the LD coefficient alone to judge image quality seems to be doubtful.

From our experience synthesized FC images with 2 subbands or more and no overlap of the bands show a significant loss in resolution and therefore may not be accepted for clinical diagnosis. However, specialized applications such as tissue characterization algorithms may benefit from the improvement in SNR.

Further we found that images compounded from multiple subbands with significant overlap enhance the subjective impression of the observer even though the amount of speckle reduction may not be optimal in the sense of maximum lesion detectability.

Providing the user with the control over the loss in resolution of the compounded image as we have pursued with our investigations seems to be a reasonable approach to overcome the problems that are related to image quality assessment.

CONCLUSION

It has been shown in this paper that the correlation curves for both FC and SC can be approximated by a Gaussian function of the kind $\rho(\Delta x) = \exp(-\Delta x^2/\alpha)$, where α is approximately 0.1 for SC and approximately 0.27 for FC. The model predicts an optimum improvement of lesion detectability or, correspondingly, an improvement in SNR for FC with two subbands up to a resolution loss of approximately 2. This corresponds to a -6 dB intersection of the subband spectra. For spatial compounding with two subapertures we find a maximum lesion detectability up to resolution loss of approximately 1.7 which corresponds to a relative aperture shift of 0.7.

For FC the predicted results were compared with experimental data and an excellent agreement was found within the relative error of the measurements.

REFERENCES

[1] Melton HE, Magnin PA: A-Mode Speckle Reduction with compound frequencies and compound bandwidths. Ultrasonic Imaging 6, 159-173, 1984

[2] O'Donnell M, Silverstein SD: Optimum Displacement for Compound Image Generation in Medical Ultrasound. IEEE Trans Ultras Ferro Freq Control, Vol. 35, No. 4, 470-476, July 1988

[3] Smith SW, Wagner RF, Sandrik JM, Lopez H: Low contrast detectability and contrast/detail analysis in medical ultrasound. IEEE Trans Sonics Ultrason, Vol SU-30, No 3, 164-173, May 1983

[4] Trahey GE, Allison JW, Smith SW, von Ramm OT: Speckle Reduction Achievable by Spatial Compounding and Frequency Compounding. Experimental Results and Implications for Target Detectability. SPIE Vol. 768 International Symp on Pattern Recognition and Acoustical Imaging, 185-192, 1987

[5] Trahey GE, Smith SW, Ramm OT: Speckle Pattern Correlation with Lateral Aperture Translation: Experimental Results and Implications for Spatial Compounding. IEEE Trans UFFC, Vol UFFC-33, No 3, 257-264, May 1986

[6] Verhoeven JTM, Thijsen JM: Improvement of lesion detectability by speckle reduction filtering: a quantitative study. Ultrasonic Img 15, 181-204, 1993

[7] Wagner RF, Insana MF, Smith SW: Fundamental Correlation Lengths of Coherent Speckle in Medical Ultrasonic Images. IEEE Trans UFFC, Vol 3, No 1, 34-44, Jan 1988